12.6: Intro to 3D surfaces

Goal: 7 basic 3D shapes and names

- Cylinders, Cones, Ellipsoids
- Paraboloids (two types) ← circular Hyperbolic
- Hyperboloids (two types) ← one sheet two sheets cone

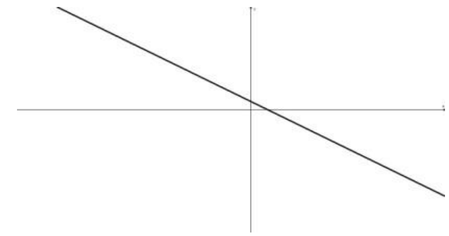
Entry Task: What are the names of these **2D curves**?

1. $3x + 2y = 1$
2. $3x^2 - y = 4$
3. $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$
4. $\frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$

A 2D curve review

Lines: $ax + by = c$

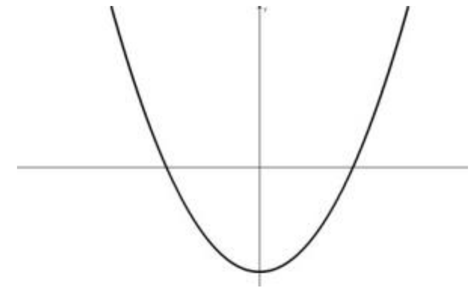
everything to first power



Parabolas: $ax^2 + by = c$ or

$ax + by^2 = c$

one is squared

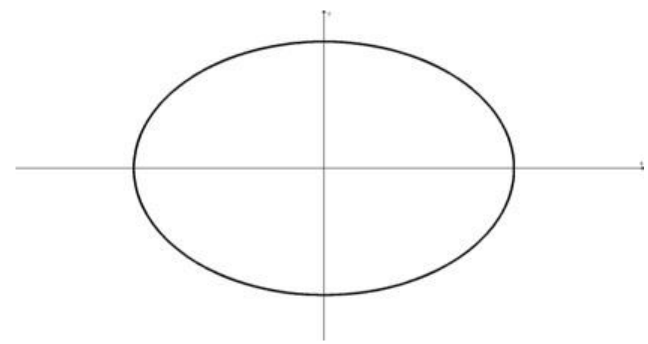


Ellipse: $ax^2 + by^2 = c$ (if $a,b,c > 0$)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(*Note:* If a = b, then it's a circle)

Both squared

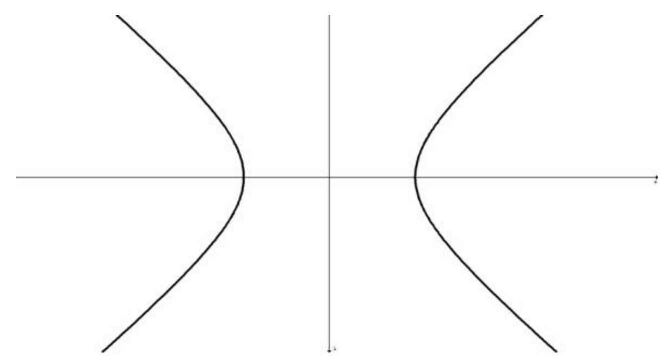


Hyperbola: $ax^2 - by^2 = c$ or

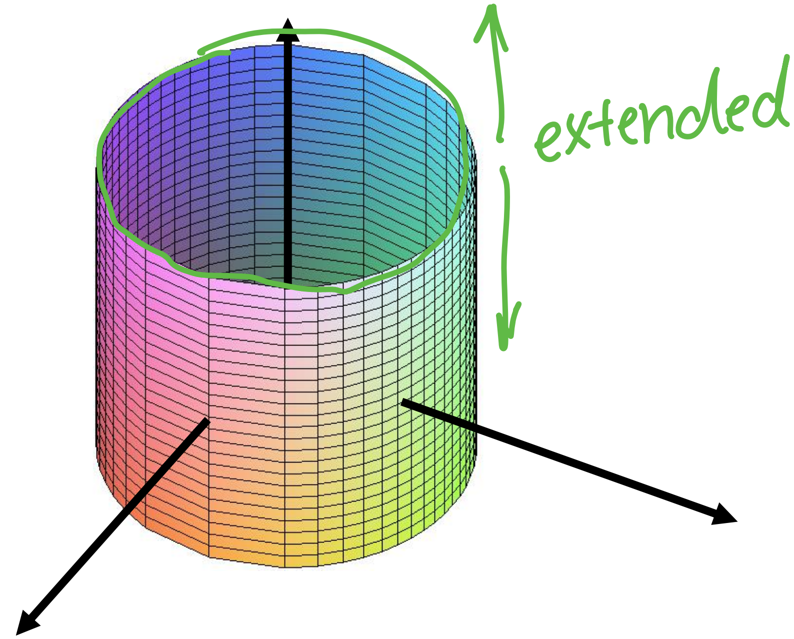
$-ax^2 + by^2 = c$ (if $a,b,c > 0$)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Both squared



Cylinders: If *one variable is absent*, then the graph is a 2D curve extended into 3D.



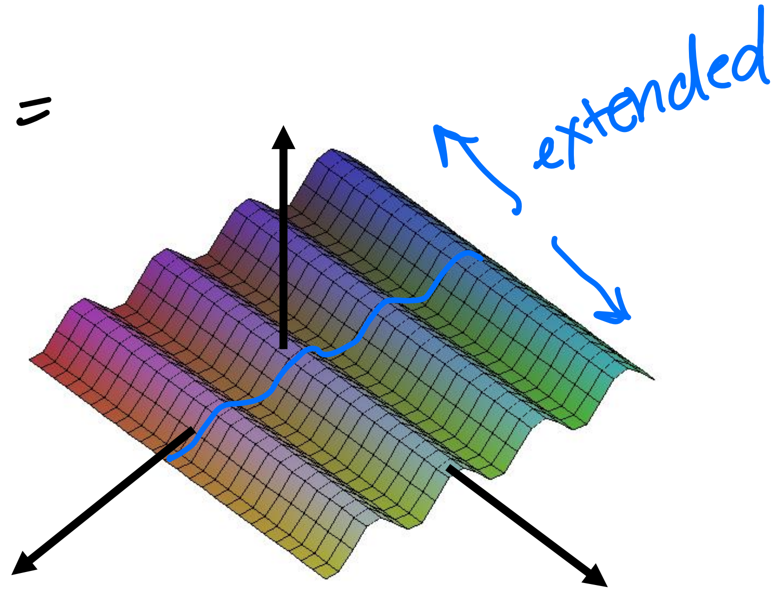
If the 2D shade is called “BLAH”, then the 3D shade is called a “BLAH cylinder”.

Examples:

(a) $x^2 + y^2 = 1$ in 3D is a **circular cylinder** (*i.e.* a circle extended in the *z*-axis direction).

cylinder = anything extended

(b) $z = \cos(x)$ in 3D is a **cosine cylinder** (i.e. the cosine function extended in the *y*-axis direction).



Quadric Surfaces: A surface given by an equation involving a sum of first and second powers of x, y, and z is called a *quadric surface*.

To visualize, we use **traces**.
We fix one variable and look at the resulting 2D picture.

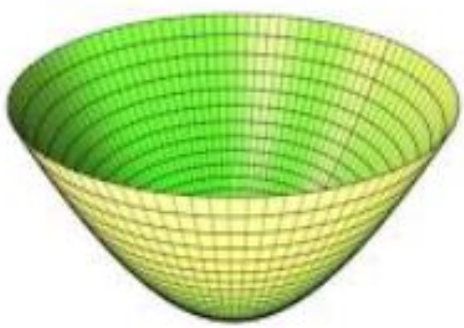
Paraboloid

$z = 3x^2 + 5y^2$

$z = 5y^2 \rightarrow$ when x is fixed, parabola in 2D

$z = 3x^2 \rightarrow$ when y is fixed, parabola in 2D

When <u>two</u> variables are squared....



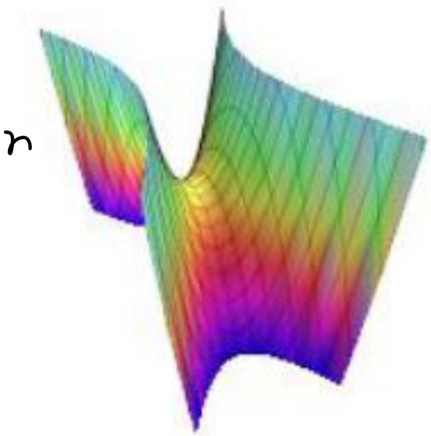
$3x^2 + 5y^2 \rightarrow$ when z is fixed, ellipse in 2D

Elliptical/Circular Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

(*ex*: $z = 3x^2 + 5y^2$)

if hyperbola in 2D



Hyperbolic Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

(*ex*: $y = 2x^2 - 5z^2$)

Example: *Spring 2011 Exam 1 – Loveless*

Consider $z = x^2 + 2y^2$.

1. Desribe the traces (2D curves) when:
   - x = k is fixed

     Parabola
   - y = k is fixed

     parabola
   - z = k is fixed

     ellipse

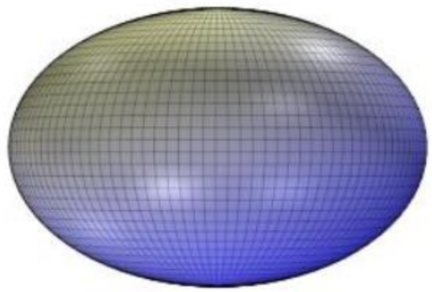
2. Give the name of this shape.

   elliptical paraboloid

visual: https://www.math3d.org/tQkmmO1X

When all <u>three</u> variables are squared....

all positive

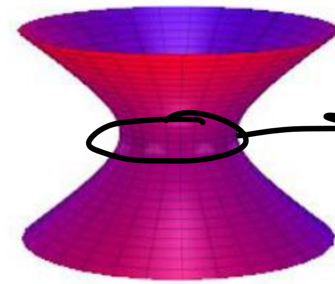


Ellipsoid/Sphere

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

(*ex*: $3x^2 + 5y^2 + z^2 = 3$)

one negative

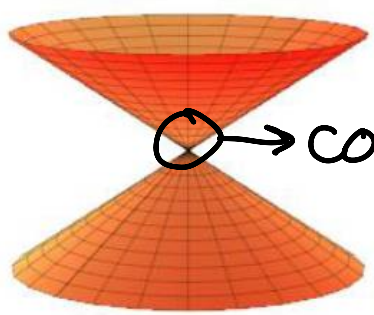


circle

Hyperboloid of One Sheet

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$ Positive

(*ex*: $x^2 - y^2 + z^2 = 10$)

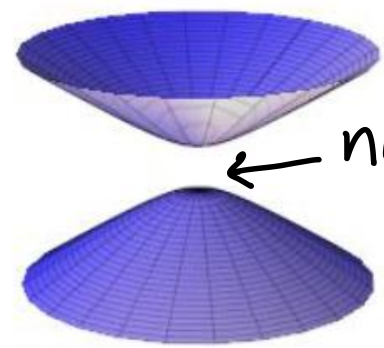


cone

Circular/Elliptical Cone

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$ zero

(*ex*: $z^2 = x^2 + y^2$)



no circle @ z=0

Hyperboloid of Two Sheets

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$ negative

(*ex*: $x^2 + y^2 - z^2 = -4$)

Hypboloids/Cone Visual: https://www.math3d.org/1viU60az

Example:

Winter 2016 Exam 1 - Loveless

Consider $(z^2 - 6x^2 - 6y^2 = -9)$ -1

Give the precise name of this shape.

$-z^2 + 6x^2 + 6y^2 = 9$

$-z^2 + 6y^2 = 9 \to$ Hyp

$-z^2 + 6x^2 = 9 \to$ Hyp

$6x^2 + 6y^2 = 9 \to$ circle

Hyperboloid of one sheet

(around z-axis)

Follow up questions:

- What happens if the locations of z and x are flipped in the equation?

$-1(x^2 - 6z^2 - 6y^2 = -9)$

$-x^2 + 6z^2 + 6y^2 = 9$

$\hookrightarrow$ around the x-axis

- What happens if we add 4z to the left side?

$\underbrace{z^2 + 4z}_{\text{CTS!}} - 6x^2 - 6y^2 = -9$

$-1((z+2)^2 - 6x^2 - 6y^2 = -9)$

$-(z+2)^2 + 6x^2 + 6y^2 = 9$

$\hookrightarrow$ shifted down 2

Visuals: https://www.math3d.org/1yedHRj3

Find the traces and name the shapes:

1) $x-3y^2+2z^2=0$

(standard form)

$x=3y^2-2z^2$

$\boxed{\text{hyperbolic Paraboloid}}$

$k=3y^2-2z^2 \rightarrow$ hyperbola

$x=3k^2-2z^2 \rightarrow$ parabola

(traces)

$x=3y^2-2k^2 \rightarrow$ parabola

2) $4x^2+3y^2=10$

one variable missing $\rightarrow$ cylinder

$\boxed{\text{elliptical cylinder}}$

3) $(5x^2-y^2-z^2=4)-1$

$-5x^2+y^2+z^2=-4$

$\boxed{\text{hyperboloid of 2 sheets}}$